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Financial Contagion and Network Structure: Simulations of Contagion in Banking Networks

Seminar Paper in Financial Economics

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α	Degree of power of Power-Law distribution
γ	Share of equity to total assets
E	Total External Assets
θ	Share of interbank assets to total assets
k	Number of nearest neighbors in small-world network
λ	Parameter of Poisson distribution
m	Number of out-going edges per node in scale-free network
n	Number of nodes in a network
p^{random}	Erdös-Rényi probability
p^{small}	Rewiring probability in small-world networks

1. Introduction

The stability of the financial system is not only since the last financial crisis 2007 of interest for many researchers and regulators. Besides the aspect of studying and understanding the behavior of individual banks, a growing number of research is conducted in the field of modelling the interbank linkages. Especially with respect to their contribution to systemic risk in the financial system. The financial system can be characterized as a very complex network of multidimensional linkages between financial institutions. While the interconnectedness provides advantages with respect to efficient pricing and provisioning liquidity, in times of crisis or financial distress the linkages can turn out to be the Achilles tendon of the financial system. A shock at first effecting only one bank can spread by multiple channels to other financial institutions and under some circumstances pose a risk to the stability of the financial system. Since the financial crisis, understanding the banking network became more important for regulators to assess systemic risk on a macro-level. Analyzing the banking network can yield important insights into the inner workings and may help introduce more effective regulations to prevent future systemic failure.

This paper gives an overview of the developments in economic and network related literature concerned with interbank markets and contagion effects (section 2). The main part is contributed to the paper from Nier et al. (2007) who conducted a simulation study on contagion effects in random graph networks. This paper applies their framework and algorithm to different network topologies in order assess the impact of network properties on contagion effects (section 3 and 4). Further model extensions which introduce additional channels of contagion (Cifuentes et al., 2005) are discussed in section 5. Section 6 summarizes the main results.

2. Related literature

2.1 Economic implications of the interbank market

Commercial banks in the financial system can be very heterogeneous in their characteristics. For example, due to different business models (savings banks vs. investment banks) or banks being in different geographical regions, banks have differing needs for liquidity. Some institutions may have a constant liquidity inflow due to increasing customer deposits, while lacking investment opportunities, which leads to excess liquidity. Whilst other banks may need liquidity to finance investment opportunities. The interbank market then takes the important role the redistribute the liquidity to where it’s needed. It is important to note that this process is only redistribution of a liquidity, no additional liquidity is created at the macro level. In normal times these mechanisms are "good" in the sense that they allow for an efficient distribution of risks and liquidity. They also may enforce peer-monitoring among the financial institutions and enhance market discipline.[1]

But the linkages between the financial institutions also can pose risks to the financial system. Based on the model for bank runs from Diamond and Dybvig (1983) Allen and Gale (2000) show, that the interbank linkages can spread a shock from a single institution to other banks leading to the possible failure of the whole financial system. A main result of their study is that the network structure is very important for the resilience of the financial system to shocks. A complete network, where every bank is connected to every other bank turns out to be very stable, since initial shocks are shared between all institutions and therefore the individual bank only has to bear a small loss.[2] On the other hand, incomplete networks can be fragile as well, since the shock is not divided into small pieces and leads to larger losses for connected institutions.

The financial crisis 2007 originated to a large part in the financial system. This spurred extended interest in the workings of banking networks and the emergence of systemic risk from them. Since common economic theory often uses general equilibrium models with representative agents, which do not account for the linkages between institutions and the resulting network dynamics, researchers started to implement techniques from network theory (Haldane and May, 2011, p. 351).

In general shocks can spread through different channels in a banking network. Early research in this field concentrated on the direct bilateral exposures, where shocks are transmitted by not fully repaid interbank loans. This direct contagion can lead to domino effects and systemic failure. These effects are for example studied in Eboli (2013) and Nier et al. (2007). But systemic failure can also stem from other channels of contagion. Another important one is the indirect channel of portfolio overlaps, meaning the exposure to common risk. A bank with a liquidity shortage may sell its assets, the following decline in asset prices can leads to further losses for all banks. Other banks may also be forced to sell assets and a fire sale scenario may emerge. Cifuentes et al. (2005) and Nier et al. (2007) among others take these liquidity effects into account. More recent studies on contagion in financial networks also study the effects of liquidity hoarding, which simulates a funding shock and affects the liabilities of a bank (Gai and Kapadia, 2010; Arinaminpathy et al., 2012).

[1] Flannery, 1996, p. 821; Rochet and Tirole, 1996, p. 735f.; Allen and Gale, 2000, p. 3.

[2] This property of complete networks depends on the structure of the balance sheets of the individual banks and the size of the shock. If the banks have very low equity buffers even minimal shocks can lead to a system wide default. Then less connected networks or at best a separated network structure may prevent a system wide default (Acemoglu, 2015).

2.2 Networks and simulation studies

Network theory and its applications developed outside of economics and only recently the dynamics of the banking networks got more attention. This section will give a short overview of the developments in network theory related to banking networks. A more technical assessment of the different network models is given in the appendix A.1.

There are two branches of papers concerned with contagion in interbank networks. One uses empirical network data to conduct analysis, the other branch bases its simulations on artificial networks. The empirical approach provides a more country specific assessment of contagion risks. Data on real-world banking networks is scarce and the obtained results may be driven by peculiarities of the individual networks. Artificial networks as an alternative can be generated at will and also allow for the study of the effect of different network parameters (Nier et al., 2007, p. 2037). This paper concentrates on simulation studies for artificial networks.[3] The literature branch using artificial networks itself can be categorized by the type of artificial networks used and the different types of contagion effects taken into account. One type of artificial network is the random graph, where the connections between nodes are determined by an independent probability, often called Erdös-Rényi probability (Newman, 2018, p. 345). For banking networks directed connections between the nodes are used, since the direction of the money flows is important when determining the contagion effects. Many research papers base their analysis on these type of networks or on modifications of it. For example, Cifuentes et al. (2005) analyze the contagion effects from liquidity risks under capital requirements. Nier et al. (2007) simulate the effects of direct bilateral exposures based on random graphs with varying Erdös-Rényi probabilities. Later studies extended the scope of the research in terms of contagion effects taken into account (see previous subsection), but also with respect to the network topologies the simulations are based on. While being a simple and accessible network model, the random graph has some flaws in the way that it characteristics differ from the ones often observed in the real world (Newman, 2018, p. 345). Two other widely known and used models are the "small-world" network (Watts and Strogatz, 1998) and the scale-free network (Barabasi and Albert, 1999). Examples can be found in Battiston et al. (2012) who used random regular networks to simulate the effects of connectivity on systemic failure and Georg (2013) who simulates a dynamic multi-agent banking system for all three mentioned type of networks. Also Eboli (2013) analyses direct contagion and provides analytical thresholds for different network topologies. Further provide Amini et al. (2016) analytical results for random graphs varying degree distributions and inhomogeneous weighted connections.

[3] A survey of the empirical research can found in Upper (2011).

3. Model Framework

This section will describe the setup of the simulation study. The framework consists of three elements: the banking network, the bank's balance sheets and the algorithm to resolve the shock. The simulation study is based on Nier et al. (2007) and extended with different network topologies.

3.1 Banking network

Following Eboli (2013) a banking network can be easily represented by a network in which banks are represented as vertices and their interbank linkages as edges. Since we are modelling the interbank market an edge corresponds to an interbank loan. The direction of the edges is important, since interbank loans are always granted from one bank to another. Beyond this basic idea, the actual methodology of creating the network and which properties it inherits varies with the different concepts of random networks. The three network topologies used in this paper are discussed in detail in the appendix A.1.

3.2 Balance sheets

For any generated network the corresponding balance sheets are calculated consistent with bank-level and aggregate balance sheet identities. Nier et al. (2007, p. 2039) describe the process in detail, here only the main properties will be presented. The balance sheet of every bank consists of the positions shown in Figure 1. Additionally, to the network structure three parameters are needed to calculate the individual balance sheets: The total amount of external assets in the financial system, the fraction of total interbank loans to total external assets and the fraction of equity to assets. Further, it is assumed that all interbank loans have the same size.

Assets	Liabilities
External Assets	Equity
	Deposits
Loans to banks	Loans from banks

Figure 1: Schematic balance sheet representation.

After defining these parameters, the individual positions for each bank can be calculated. Using the network structure and the size of the individual interbank loan the positions "loans to banks" and "loans from banks" can be set. Afterwards the amount of external assets is calculated in two steps. First setting the individual external assets equal to net interbank lending for each bank and in a second step distributing the remaining external assets equally across all banks. Now the assets side of the balance sheet is complete and subsequently the equity can be calculated as a share of assets. The customer deposits are calculated as a residual value.

3.3 Simulation and contagion algorithm

The contagion algorithm describes the inner workings of the banking network when a shock occurs. The procedure consist of mechanical balance sheet calculations after a shock has happened. There is no further stochastic or dynamic behavior involved, the stochastic behavior of this simulation study solely stems from the network generating process. The contagion process starts with a shock to an individual bank. The size of the shock can be varied; in the baseline scenario all external assets of the shocked bank are lost. Since the liabilities of the bank underlie a priority when handling losses first the equity of the banks is used to cover the occurred loss. If the equity is equal or smaller than the loss, the bank is considered to be bankrupt. When the loss is greater than the equity the remaining loss is applied to the received interbank loans.[4] Depending on the size of the loss the bank defaults either partially or completely on the interbank loans. In the case of a partial default the losses are distributed equally among the received interbank loans since they all have the same size. In this step the direct contagion takes place and losses get passed on via the interbank market. In the case of a complete default on the received interbank loans the remaining losses are covered by the customer deposits.[5] In the next iterative step, the losses transmitted by the interbank market get applied to the affected banks and the procedure starts again. The procedure ends when no further banks default or all banks in the network have defaulted. The shock procedure gets applied to each bank in the network and the average number of defaulted banks is saved as a measure of system fragility. Since the networks stem from a random process the contagion algorithm is applied to 100 random networks for each parameter combination in a Monte Carlo simulation to study the average behavior of the networks with respect to different parameter combinations. The results from these simulations are discussed in the following section.

4. Simulation results for different network types

The structure of the artificial banking system can be described by four to six parameters depending on the chosen network model. Table 1 in the appendix gives a brief overview of the baseline parameters. Each parameter will be varied according to defined range of variation in Table 1. The results for the random graph networks are a replication of the results from Nier et al. (2007). After the

[4] Assuming the bank under consideration has received interbank loans.

[5] In the sense of a flow network the customer deposits act as sink which receives the flow originating from the source (the shock).

replication the same contagion algorithm is applied to small-world and scale-free networks to illustrate the influence of the network topologies. All presented figures show as solid lines the average number of (relative) defaults over 100 network realizations for a given parameter combination.[6]

4.1 Bank capitalization

The first parameter to investigate is the share of equity to total assets, hence the equity base of the banks in the system. For random graph networks Figure 2 shows a monotonically decreasing relationship between the number of defaulted banks and the share of equity. Since a higher share of equity means that each affected bank can absorb more of the applied shock and less losses are passed on to connected banks. It is to note that the relationship is nonlinear, between 0% and 2% share of equity we see a sharp decline in the number of defaults. From a complete default of the system for very low levels of bank capitalization to only around 6 defaulted banks. In the range of 2% to 3.5% the number of defaults does not change with the increasing amounts of equity. For higher amounts of equity relative to assets the number of defaults converges to one, only the initially shocked bank defaults due to the large shock.

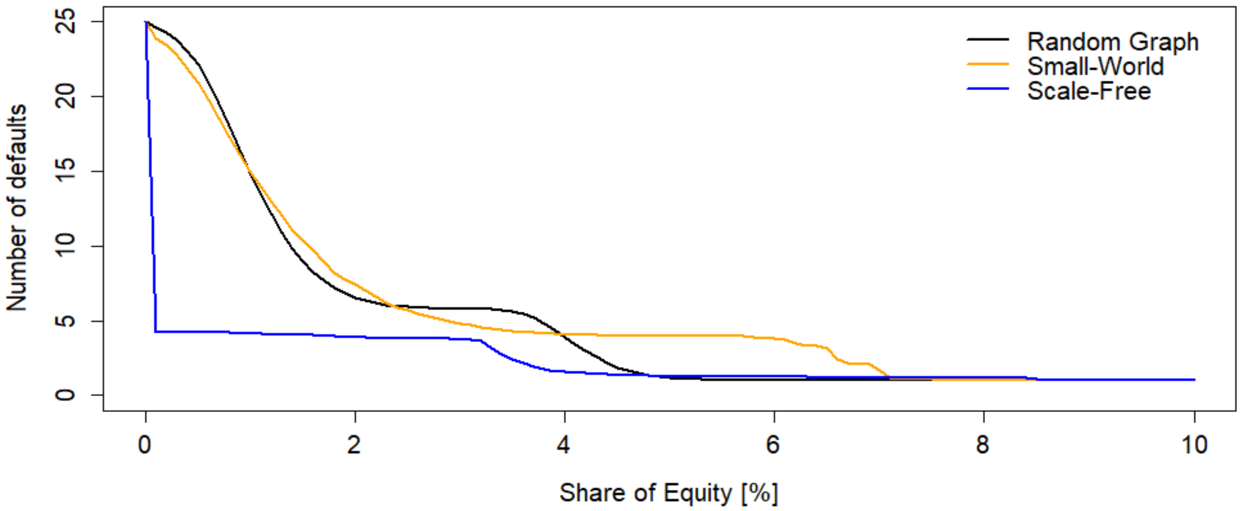

Figure 2: Average number of defaults in relationship to the share of equity to total assets. Following Nier et al. (2007) Fig. 1.

The simulation results for small-world banking systems look similar in general. We also see a monotonically decreasing relationship, which is nonlinear. But the initial decline of defaulted banks to an increased share of equity is not as sharp as for random graphs. Also a stable socket of 5 defaults is only reached with an equity share of 4%. Further the socket seems to be present for a longer interval, since only for an equity share of 6% and higher the number of defaults converges to one.

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[6] Shaded areas in some figures represent the range in which 95% of the observations lie. Figures in which all three network topologies are combined do not include these shaded areas for visual reasons, but can be found in the appendix A.3.

Another difference is, that the observed results show less variation especially around the last decline in the number of defaults (Figure 11). The reason for the higher number of defaults should lie in the increased clustering and connectedness of the small-world network. This way the shock affects more banks in less rounds of the shock procedure. This means that more of the initial shock gets transmitted to other banks and is not absorbed by customer deposits like in the random graphs.

The results for the scale-free network show a significantly different behavior, although a monotonically decreasing relationship is also present. A default of all banks is only observed for shares of equity of effectively 0%, even for slightly higher values the average number of defaults drops to only 4 banks. Up to 3% this value decreases only by a marginal amount, then for higher values of bank capitalization we see the typical convergence to one default. But the value of exact one default is only reached for equity shares of 8.5% and higher. These results can be explained by the preferential attachment mechanism of the scale-free networks, which lead to the presence of central nodes to which all other nodes have connections. Since the shock applied to the system is dependent on the amount of external assets a banks holds which follows from its net interbank lending. If a central node is shocked, the shock applied to the system is significantly larger in comparison to a shock resonating from a peripheral node. In the case one of the central nodes is shocked the whole or a large part of the system fails, if on the other hand a peripheral node is shocked on the initially shocked node fails.

The simulations show that the capitalization of banks plays a crucial role with respect to system stability and limiting contagion effects. The different network topologies have in general similar results, nonetheless each topology requires different levels of equity to prevent widespread system failure.

4.2 Interbank assets

In a second round of simulations the influence of the share of interbank assets to total assets is analyzed. For the random graph model one finds only the initially shocked bank defaulting for interbank asset shares of up to 20%. It follows a rather fast increase of the average number of defaults until the effect levels off at around 6 to 7 defaults for interbank assets shares of 30% and more (Figure 3). This behavior can be attributed to the construction of the balance sheets. The amount of total external assets in the system is fixed, since the total amount of interbank assets is calculated as a share of the external assets[7] an increase in interbank assets leads to an increase of total assets. Further the amount of equity is defined as a share of total assets, by increasing the interbank assets

[7] Therefore, also implicitly as a share of total assets.

also the amount of equity increases. While the shock size defined as amount of external assets a bank holds stays the same. This leads to two opposing effects: First the increase in interbank assets increases the amount of losses that can be spread by contagion to connected banks. Second the amount of losses that is spread gets first absorbed by a better capitalization of the banks.

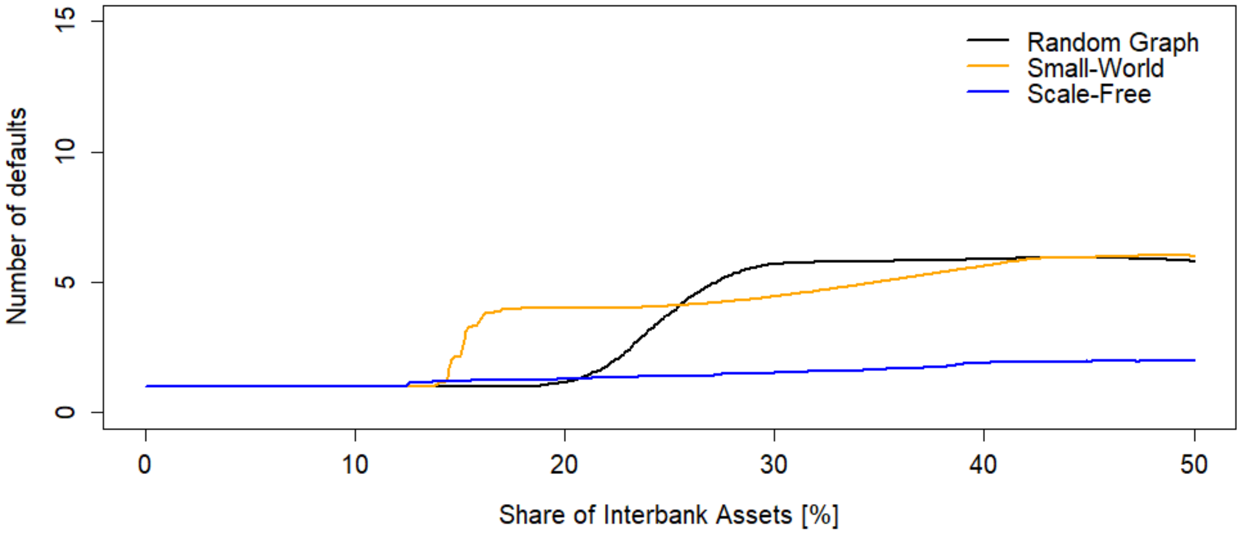

Figure 3: Average number of defaults in relationship to the share of interbank to total assets. Following Nier et al. (2007) Fig. 2.

Above the 20% threshold the amount of losses that are spread by contagion are now large enough to cause second round defaults. At the second threshold of 30% the bank capitalizations have improved enough to limit the second and higher round defaults to 6 to 7 banks.

The opposing effects originating from the balance sheet construction are identical in the case of the small-world and scale-free network models. For the small-world networks we see a sharp increase in defaults already at around 15% share of interbank assets. It follows a slow nonlinear increase to approximately the same level of defaults as in the random graph. Small-world networks seem to be more sensitive with respect to the size of the interbank assets, since passing the threshold leads to a sharp increase in defaults. The higher clustering coefficient and the increased connectedness (regularity of three out-going connections per node) of the small-world model leads to a faster spreading of the losses and when the threshold is passed this leads to the observed level of defaults.

Simulation results based on scale-free networks differ significantly from the previous ones. Although again a threshold value of this time roughly 12.5% can be found, crossing this value only starts a very slow increase in defaults, which does not reach the average default levels of the random graph and small-world networks. Instead the average number of defaults only increases up to 2 to 3 defaults. This is attributable to the fact that the equity share seems large enough to absorb contagion stemming from the majority of the small banks in the network. While the contagion and default of the large banks does not show in this average results since only 2 to 3 banks emerge in this setup as large banks. Analyzing the individual results, shocks to the large banks (central nodes) lead often to a complete system default.

In summary increasing the share of the interbank assets leads across all network topologies to an increase of the number of defaults, but the effect levels off, if due to the balance sheet mechanics the amount of equity has increased enough to limit losses caused by contagion.

4.3 Concentration

Further analysis can be undertaken with respect to the concentration of networks, meaning varying the number of banks in the network while holding the amount of external assets fixed. The reduction of banks in the system leads to an increase in the amount of external assets the individual banks hold and therefore a full wipe out of external assets poses a larger shock to the system. To compare how different sized networks, behave the shock size is also left as a variable in this part of the simulation. First across all network topologies an increase in the shock size leads to a higher percentage of defaults, although this effect levels off for relative shock sizes larger than 40% (Figure 4).

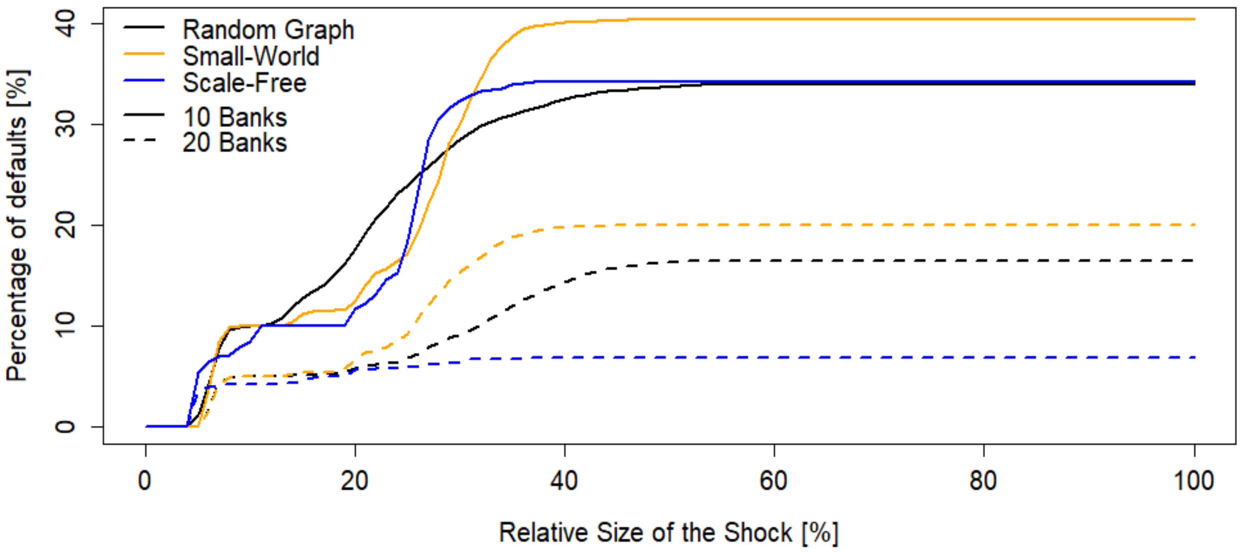

Figure 4: Average percentage of defaults in relationship to the relative shocks size for a banking network consisting of 10 and 20 banks. Following Nier et al. (2007) Fig. 4.

Another effect that is similar across the tested topologies is that a more concentrated network is object to a higher percentage of defaults compared to a network where the same amount of external assets is spread across more banks. The fact that more concentrated systems are object to a higher percentage of defaults is not only rooted in the increase of the shock size. Additionally, the individual size of the interbank loans increases in concentrated networks therefore a larger amount of the initial shock can be spread.

With respect to the differences between the network topologies it is to note that the scale-free networks seem to be particularly vulnerable to concentration. Since a network of 20 banks seems to be relative independent to the shock size while the more concentrated network quickly shows an increase in the defaults for higher relative shock sizes. The reason may be, that in a concentrated network the central nodes play an even more important role, especially when evaluating average defaults.

4.4 Varying network parameters - connectivity

Each network topology has specific parameters which determine the structure of the realizations of these networks (Table 1). This subsection evaluates the influence of these structural parameters on the number of defaults.

For the random graph the only parameter aside the number of nodes is the Erdös-Rényi probability which determines how connected the network is. In Figure 5 the whole range of the probability is depicted for different values of equity. First one finds a conformation of the findings from Allen and Gale (2000), complete network structures are very resilient to shocks only the initially shocked bank defaults.

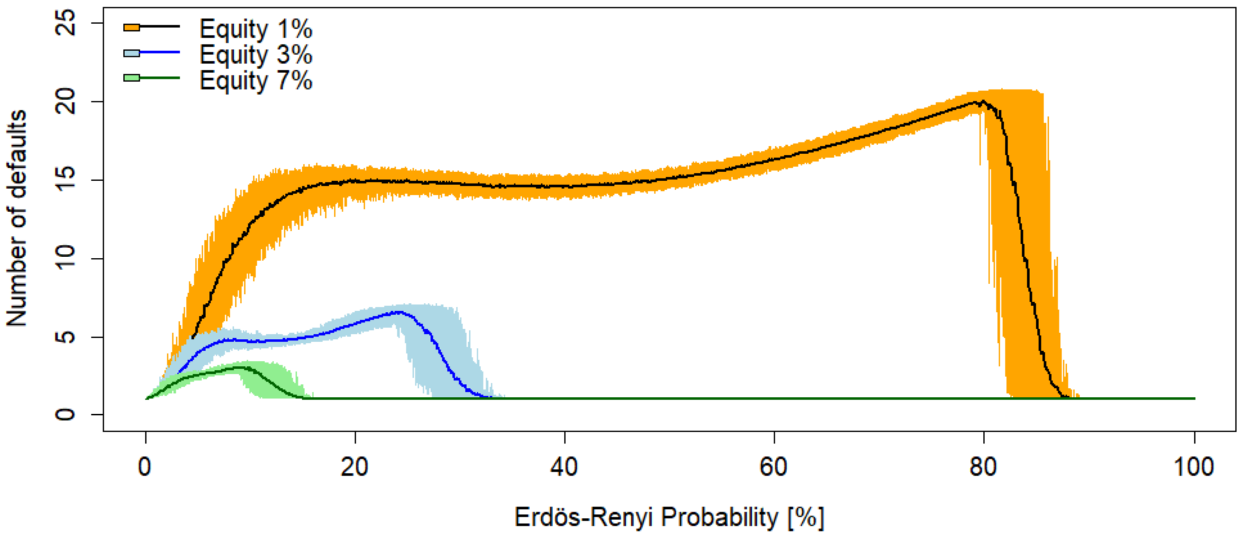

Figure 5: Average number of defaults in relationship to the Erdös-Rényi Probability. Following Nier et al. (2007) Fig. 3.

In general, Nier et al. (2007) find an M-shaped relationship between the number of connections and the number of defaults. This nonlinear relationship is owed to the circumstance that the number or likelihood of interbank connections has two opposing effects. Interbank connections are the channel of contagion but the more there are, the smaller the individual amount of spread losses becomes. Then depending on the amount of equity many interbank connections like in the case of a (near) complete network can limit the number of defaults. For undercapitalized banking systems the connection level needs to be very high to have positive effects. For only minimal higher equity shares the positive effect of a connected network sets in much earlier. This again pronounces the importance of a well-capitalized banking system, but also shows that for higher connected systems the share of equity can be reduced without increasing the number of defaults due to contagion.

For small-world networks one can vary two parameters aside from the number of nodes. One is the rewiring probability which determines with which probability an existing edge is randomly rewired. The second parameter is k the number of edges each node has. First we see in Figure 6 that there is a tendency that more regular networks (low rewiring probability) see a higher number of defaults. This is due to the circumstance that in the limiting case of a pure regular network with $k = 1$ (a circle) the shock only spreads to one connected bank in each iteration. When increasing the rewiring probability, the chance that the shocked bank has lend money from two or more other banks increases. Therefore, the shock gets transmitted to more banks at once, but it is also split up into smaller parts.

This leads to a reduction in the number of defaults. This effect is especially visible in the case that banks have initially only one outgoing and one ingoing connection ($k = 1$). For a higher number of connections $k > 1$ the number of defaults decreases.[8] Particularly networks with a high number of initial connections profit from the same effect that ensures stability in (near) complete networks of the random graph.

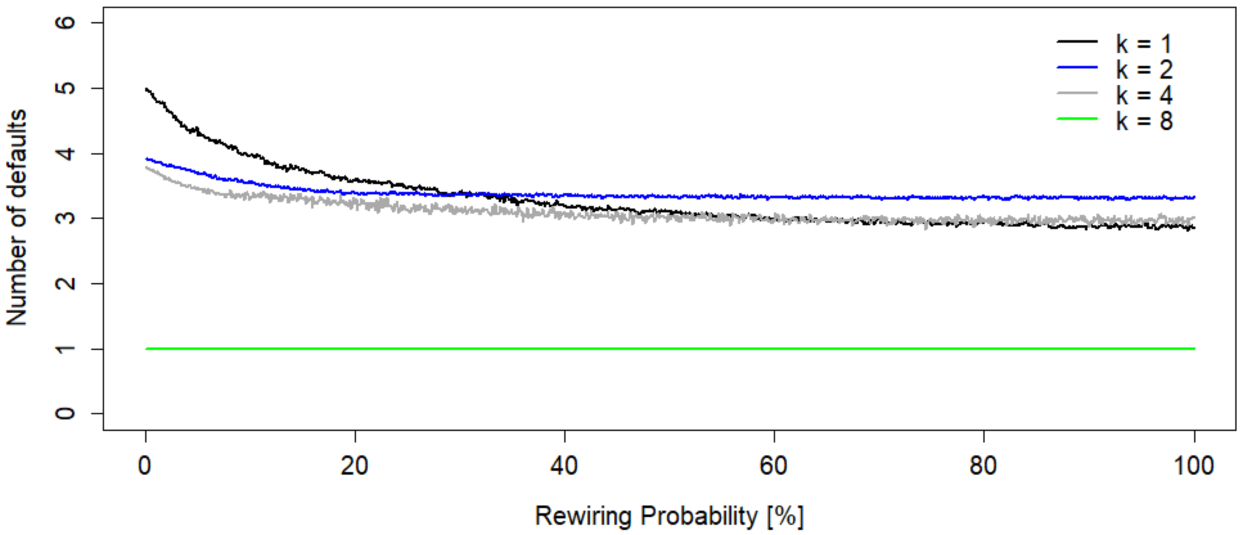

Figure 6: Average number of defaults in relationship to the rewiring probability of small-world networks.

[8] One anomaly is that for $k = 1$, the number of defaults decreases for high rewiring probabilities even under the levels of higher connected networks. This may be due to the fact that with the higher rewiring probability the chance increases to create networks which have unconnected components or isolated banks. Therefore, shocks may not be able to spread to other banks.

Scale-free networks have like small-world networks two structural parameters additional to the number of nodes. The first parameter is α which is the degree of power governing the power-law and the second one is m the number of edges each new node establishes to the existing nodes in the preferential attachment algorithm. Figure 7 shows that increasing the power of the power-law distribution decreases for small m the number of defaults.

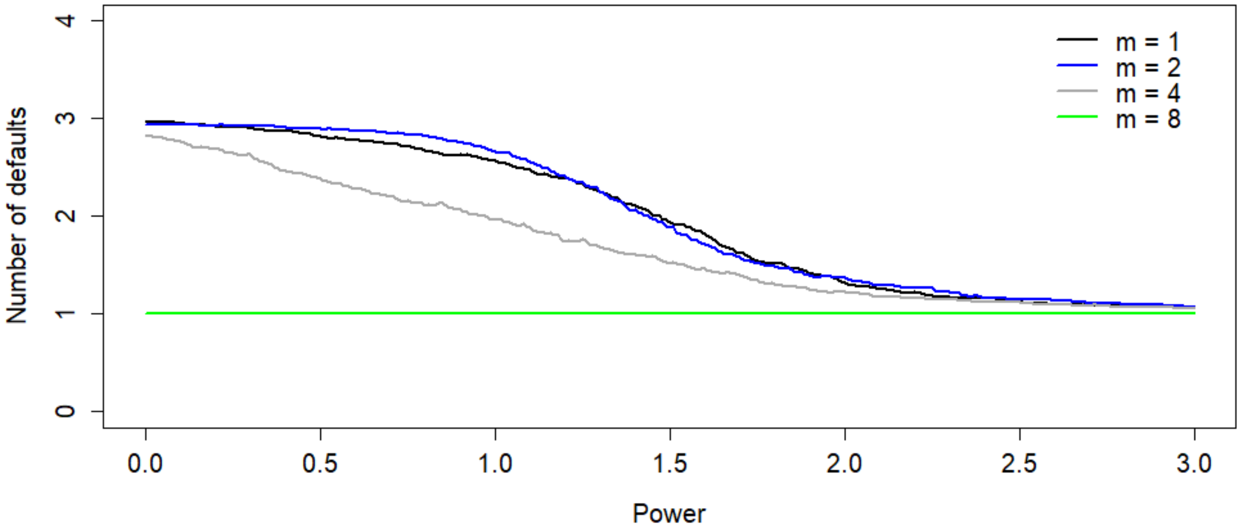

Figure 7: Average number of defaults in relationship to the degree of power of the power-law function of scale-free networks.

Since an increase in the degree of power leads to higher importance of already existing connections for forming new edges, a higher power endorses the formation of only one central node. Meaning for a banking network it emerges one single bank from which all other banks borrow money, this is also called a star shaped network. These type of networks are stable yet fragile, since a shock applied to the central node likely brings the whole system down, while a shock to the peripheral banks does not lead to a fail of any other bank (Eboli, 2013, p. 19f.). In the case of a larger amount of outgoing edges per node ($m > 1$) more than one central node emerges from the algorithm. Hence reducing the fragility of the system with respect to a shock to this central node. For highly connected systems (e.g. $m = 8$) the effect from the central nodes vanishes and the applied shocks get dispersed in small parts and lead to no second round defaults, similar to complete networks.

In summary the simulations showed that the complete network property leads to a better resilience across all tested topologies. Further all three topologies showed convergence to one average default when the network had at least ~30% of all possible connections.[9]

[9] $p^{random} \approx \frac{k}{n} \approx \frac{m}{n} \approx 30\%$

5. Extensions to the basic model

The simulation results above are based on a simple model which only takes into account the channel of direct contagion through interbank loans. Another important channel by which shocks may spread is portfolio overlaps meaning the exposure to common assets and risks. The following subsections will discuss this second channel of contagion and further extensions to the simulation studies.

5.1 Liquidity risk

Multiple studies including Cifuentes et al. (2005) and Nier et al. (2007) consider the change of asset prices as another source of contagion effects. A bank who is default has to be liquidated to pay back connected banks and customers, therefore the bank has to sell all it external assets. When doing so the price[10] of the asset will drop, not only for the defaulting bank but also for all other banks in the network who hold the same asset. Other banks holding that asset have to reevaluate their assets and cope with the losses. This can lead to further defaults and a downward facing price spiral due to fire sales. Extending the simulation algorithm above by this mechanism Nier et al. (2007), p. 2049f. find a sizeable increase in the number of defaults.[11] This is the case for all parameter combinations tested on the random graph network. The intuition is clear, since due to the decrease in assets prices the total amount of losses spreading through the system is amplified and also now not only directly connected banks experience contagion.[12] This is particularly true for concentrated or star-shaped networks, since the concentration of assets in single institutions leads in the case of a failure of these institutions to a significant asset price drop. In summary modeling the liquidity risk amplifies the effect of the initial shock and amplifies the simulation results obtained in previous the section.

5.2 Capital requirements

The model used for the simulations is still an abstract one. Cifuentes et al. (2005) change this by introducing the restriction of capital requirements which banks have to meet. They base their simulations on a banking network comparable to the one Nier et al. (2007) uses, but they do not resolve their model by a sequential default algorithm, instead they use Eisenberg and Noe's (2001) default algorithm, which accounts for the simultaneity problem when resolving networks. Also Cifuentes et al. (2005) introduce a liquid and an illiquid asset, the liquid one can be sold any time at a constant price while the illiquid one has a varying price that decreases with the amount of illiquid assets sold.

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[10] The price of the external asset is modeled as a decreasing function of the total amount of sold assets.
[11] Figures of simulation results from an extended algorithm with liquidity risk are in the appendix A.4.
[12] This especially leads to a smoother relationship between the Erdös-Rényi probability and the number of defaults, since banks no longer have to be directly connected to experience contagion. In general, the found threshold values become less pronounced.

The mandatory capital requirements force the banks to rebalance their balance sheets when losses occur due to not fully repaid interbank loans or losses from price decreases of the illiquid asset. At first banks sell their liquid asset to rebalance, but once they have no liquid assets left they are forced to sell the illiquid asset an incur further losses. The implementation of the capital requirements mainly adds another reason beside default of other banks for the banks to sell parts of their assets. Therefore, one expects it to amplify the simulation results even further. One interesting aspect of this model extension is, that the real outcome or risk stemming from one initial shock only becomes obvious after observing multiple periods. Since the initial default of one bank can trigger an at first slow process of asset selling which starts to speed up with the increasing amount of assets sold until it leads to actual defaults of banks due to the losses incurred from fallen asset prices. This is possible since no banks have to fail in order to force individual banks to sell parts of their assets, since they must oblige with the capital requirements.

5.3 Approaching reality

In recent years there were made more advances in the application of network based analyses in economic research of financial stability. Thereby the applied models take more and more aspects of the real-world into account. For example, Gai and Kapadia (2010) study the effects of liquidity hoarding. Liquidity hoarding can be seen as a shock to the liability side of a bank's balance sheet. When banks in the network stop giving out (renewing) interbank loans the funding for their counterparties dries up. This can lead in the case of no other funding sources[13] to the sale of assets and falling prices which in turn poses a threat to system stability. This extension seems especially important with respect to the causes and developments in the financial crisis 2007, where due to the sudden collapse of Lehman Brothers banks stopped their interbank lending since they were not sure if their counterparties would be solvent to repay their loans the next day.

Another missing aspect of the presented models is the aspect of the behavior of banks and how they react to losses. The shown model only uses a deterministic, mechanical seeming representation of bank behavior, it does not account for adaptive behavior. Chan-Lau (2017) presents an agent-based model approach to simulate a banking network. It allows the banks dynamic balance sheet adjustments based on risk optimization. This even allows to simulate the "natural" emergence of a banking network and observe how the banks form their connections.

[13] Other sources can be other banks or refinancing operations at the central bank for which collateral is needed.

In general, the consideration of real-world elements like heterogeneous assets (Aldasoro and Alves, 2016), creating links to the real economy (firms, consumers) or the usage of realistic networks (empirical data) can be important to better assess financial stability from a viewpoint of system risk due to contagion effects.

6. Conclusions

The simulation studies from Cifuentes et al. (2005) and Nier et al. (2007) provide some basic insights into the inner workings of banking networks. First they show that bank capitalization plays a crucial role in preventing systemic failure since equity is the first line of defense in a crisis situation. Although the simulations showed that lower equity buffers can be set off by a higher connected network, connections still are a possible channel of contagion and only true all dimensional separation provides protection from contagion effects. Further it is found that increased interbank lending also increases the number of defaults, but this effect levels off since by construction it comes with an increase of the equity buffers.

With respect to the concentration of banking networks the simulations show a too big to fail risk. Concentration of assets and interbank lending on few institutions poses in case of a failure of such institutions a large risk to financial stability. With respect to network topologies the results above showed little difference between random graph and small-world models. Scale-free networks on the other hand are specific due to the emergence of highly connected hubs from the preferential attachment algorithm. In the case of a collapse of one hub the whole system is at risk, while shocks to the peripheral banks mostly cause no extensive harm. One policy implication from this is that banks who pose an increased risk to the system should be object to tougher regulations, as argued by Haldane (2009).

The model extensions which introduced further channels of contagion amplified the before obtained results and showed that especially liquidity risk is an aspect to consider when assessing contagion effects. The capital requirements added another restraining condition to the network, which allowed for the slow buildup of a systemic failure. The results should be seen in the context that real-world networks feature multilayered and complex linkages between financial institutions. Especially the surge in financial innovations and expanding derivative markets in the last decade could add new interesting dimensions to banking network analysis.

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Appendix

A.1 Networks and simulation parameters

A.1.1 Random graph

A random graph is defined by only two parameters: the number of nodes n and the connection probability p^{random}.[14] To create a random graph an algorithm just creates the given number of nodes and determines with the defined Erdös-Rényi probability if two nodes are connected or not. An important property of each network is its in- and out-degree distribution, since it shows the proportions of nodes with a certain amount of connections. The in- and out-degree distributions of a random graph follow a Poisson distribution with mean $\lambda = n * p^{random}$ (for a large number of nodes). Which means that most nodes have the average amount of in- and out-going connections. But in real-world networks it is often observed that most nodes have way below average amount of connections, while few nodes act as hubs and are highly connected. Additionally, random graphs show a short average path lengths and a low clustering coefficient, since the connections are made purely random. In summary and conveyed to a banking network these properties mean that interbank relations are based on pure randomness and also no structure in the form of a tiered banking system or money-center hubs occurs (Newman, 2018, pp. 342-366).

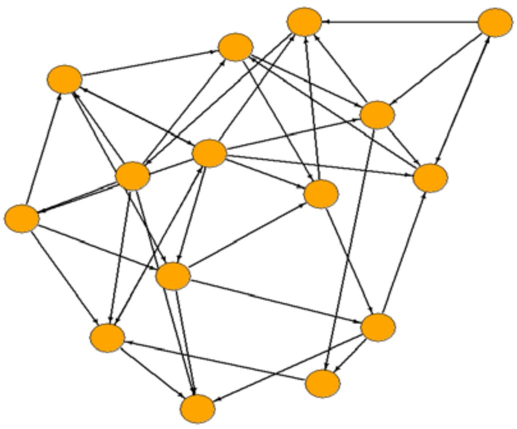

Figure 8: Example of a random graph.

[14] Alternative to this specification is to define not the connection probability but the number of edges to occur in the network (Newman, 2018, p. 343).

A.1.2 Small-world network

Another type of networks are small-world networks, which are a combination of a network build on regularity and random disorder. The small-world model starts with a circle of n nodes, where each node is connected to its k nearest neighbors. Then, each existing edge gets rewired with a probability p^{small}. A higher probability leads to more disorder in the network, $p^{small} = 1$ ultimately leads to a random graph (Figure 9). The random rewiring leads to the creation of short cuts, which allow for the small-world phenomena of being able to reach any node from any node within a small amount of steps (Watts and Strogatz, 1998, p. 440). More technically speaking the rewiring reduces the high average path length of a regular network while maintaining the high clustering coefficient. This can be particularly of interest when studying contagion, since the short paths and high clustering coefficient may lead to a higher vulnerability with respect to contagion effects. Also, if Credit Default Swaps are taken into account when analyzing interbank linkages, these networks resemble the properties of small-world networks. They seem to act like a short cut between banks (Markose et al., 2012).

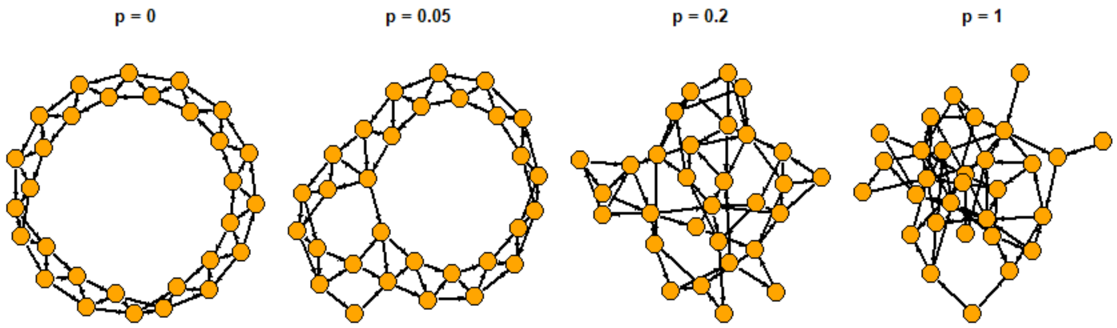

Figure 9: Comparison of small-world networks with different rewiring probabilities.

A.1.3 Scale-free network

Scale-free networks differ from the previously discussed network types. In random graphs and small-world networks the probability of a connection between two nodes is independent of any existing properties of these nodes. In contrast to that scale-free networks are based on preferential attachment and the probabilities are no longer independent of the network structure. For this an algorithm to create such a scale-free network actually simulates a "natural" growing of the network. The algorithm starts with a small number of existing nodes. Then in iterative steps adds new nodes to the network, these nodes establish m connections to existing nodes. The probability to which nodes these new edges are established is not independent, instead it depends on the number of connections the existing nodes already have (Figure 10). This growing mechanism continues until a set threshold of n nodes is reached. This preferential attachment algorithm leads to a scale-invariant state where the probability that a node has a certain amount of edges follows a power-law distribution with degree of power α (Barabasi Albert, 1999, p. 510f.). This property is often observed in real-world networks, like the world wide web or social networks. Also the degree distributions of banking networks can be fitted to power-law distributions (Soramäki et al., 2007, p. 325f.; Bech and Atalay, 2010, p. 5234f.). But the degree distributions of most real-world networks, including banking networks, does not follow the power-law distribution for small degrees. The power-law distribution mainly applies to the long right tail of the degree distribution and describes the existence of hubs. Applied to a banking network the preferential attachment resembles the fact that larger and more interconnected banks are more trustworthy and form money-center hubs of the network (Georg, 2013, p.2222).

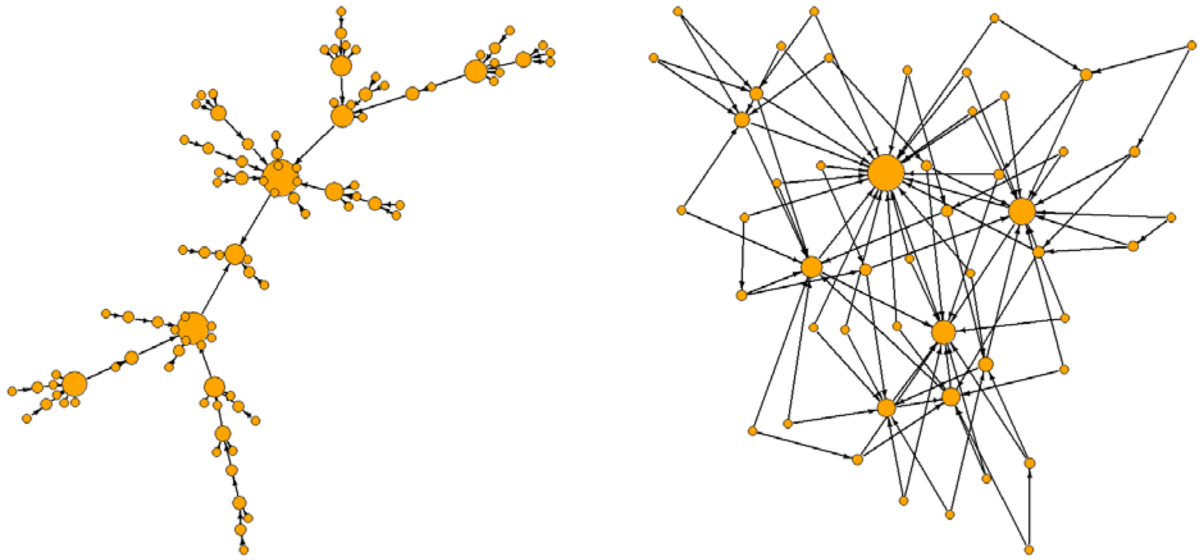

Figure 10: Comparison of different scale-free networks. Left $n = 100, m = 1$ and right $n = 50, m = 2$ with both $\alpha = 1$.

A.1.4 Simulation parameters

Parameter	Description	Baseline Scenario	Range of variation
E	Total External Assets	100,000	Fixed
n	Number of nodes in the network	25	10, 15, 20, 25
θ	Share of interbank assets to total assets	20%	0% – 50%
γ	Share of equity to total assets	5%	0% - 10%
Random Graph			
p^{random}	Erdös-Rényi Probability	20%	0% - 100%
Small-World			
p^{small}	Rewiring Probability	20%	0% - 100%
k	Number of nearest neighbors	3	1, 2, 4, 8
Scale-Free			
α	Degree of Power*	2	0 - 3
m	Number of edges each new node establishes	3	1, 2, 4, 8

Table 1: Summary of simulation parameters. General and random graph parameters are based on Nier et al. (2007) Table 1.

*Taken as approximation of real-world banking networks from Soramäki et al. (2007), p. 325.

A.2 Simulation algorithm

The simulation results presented in this paper are on a procedure written in R. The algorithm for resolving a banking network and calculating contagion effects is based on Nier et al. (2007). For the random creation of the networks was the R package "igraph" used.

The procedure for the basic model is able to reproduce the results presented by Nier et al. (2007) for the random graph model. Deviations can be seen in the simulation of the effects of the Erdös-Rényi probability, my suspicion is that these may stem from the way simultaneous defaults are handled. In my procedure when two banks default in the same iteration and they are connected the additional losses occurring to one or both of the banks are taken into account before calculating the contagion effect.

The results from the model with liquidity risk are broadly in line with the results from Nier et al. (2007) in the way that they amplify the previous results. Although the calibration of the liquidity effect is a bit unclear since Nier et al. (2007) describe values of 1 and higher for the price parameter in the price function, while actually setting the parameter to 1 would lead to an instant price decrease to zero if assets are sold. I chose instead to calibrate the price function like proposed from Cifuentes et al. (2005) such that when all external assets are sold the price has decreased by 50%, which implies a price parameter of 0.000007.

A.3 Additional figures – basic model

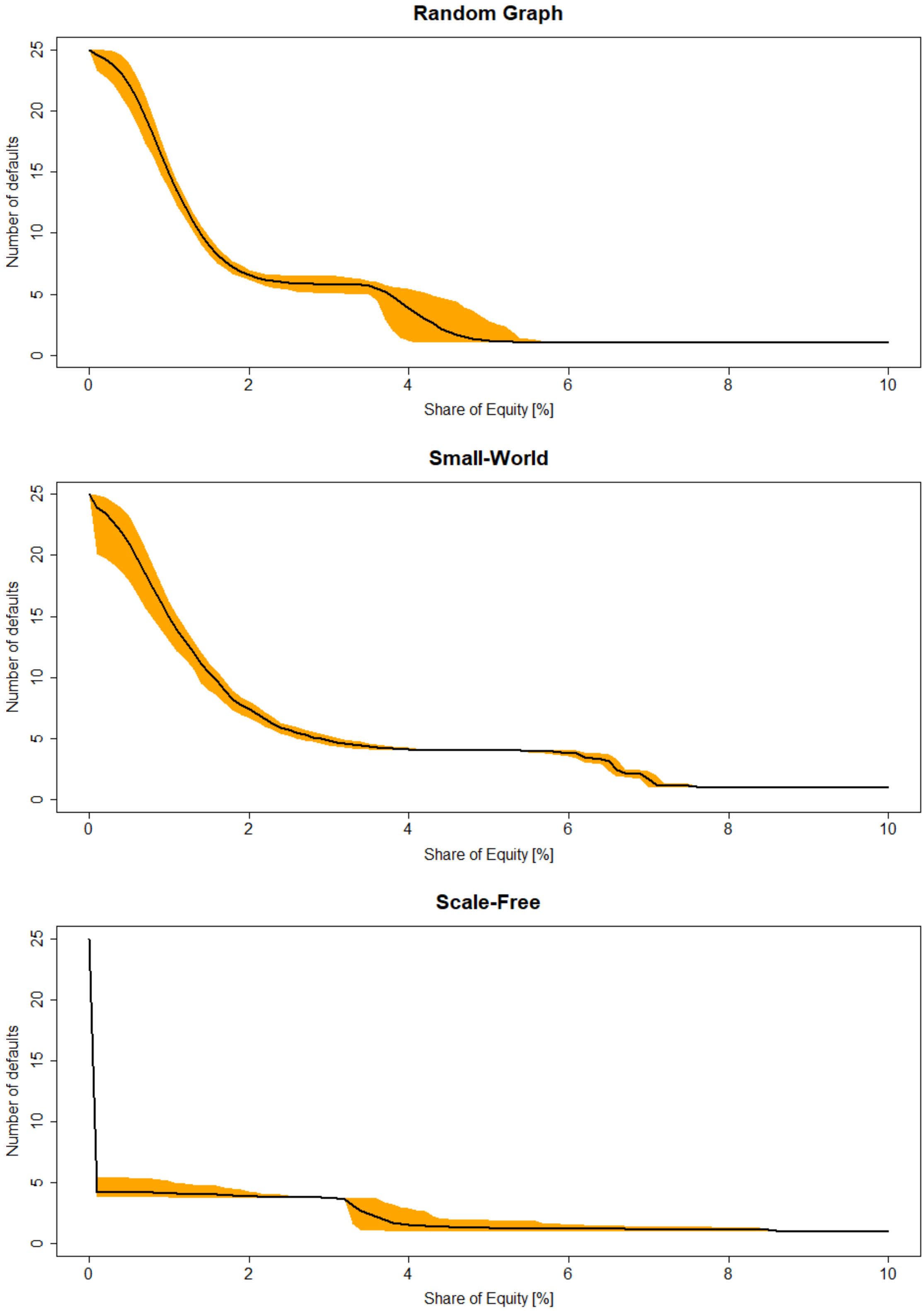

Figure 11: Average number of defaults in relationship to the share of equity to total assets for different network topologies. Following Nier et al. (2007) Fig. 1.

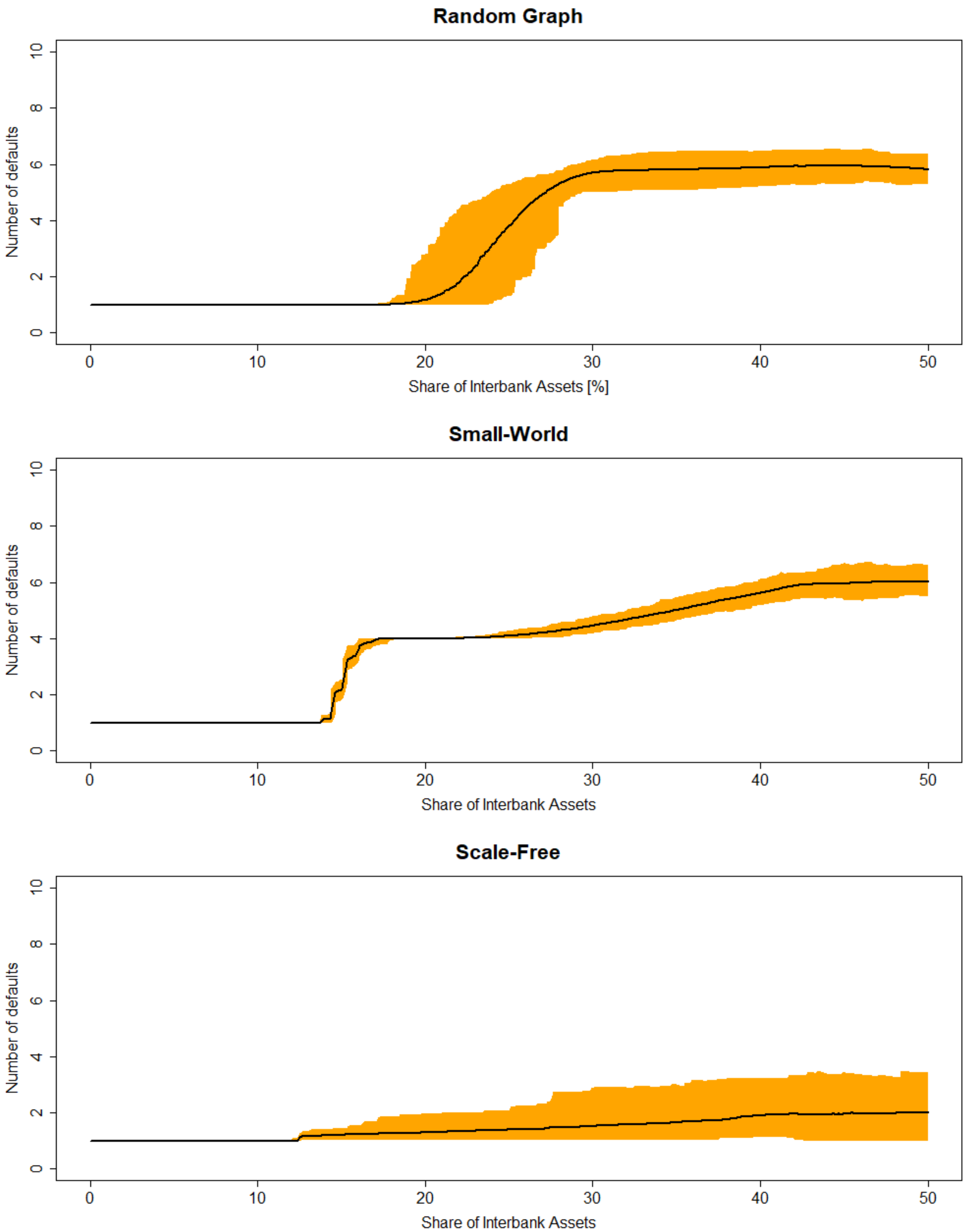

Figure 12: Average number of defaults in relationship to the share of interbank assets to total assets for different network topologies. Following Nier et al. (2007) Fig. 2.

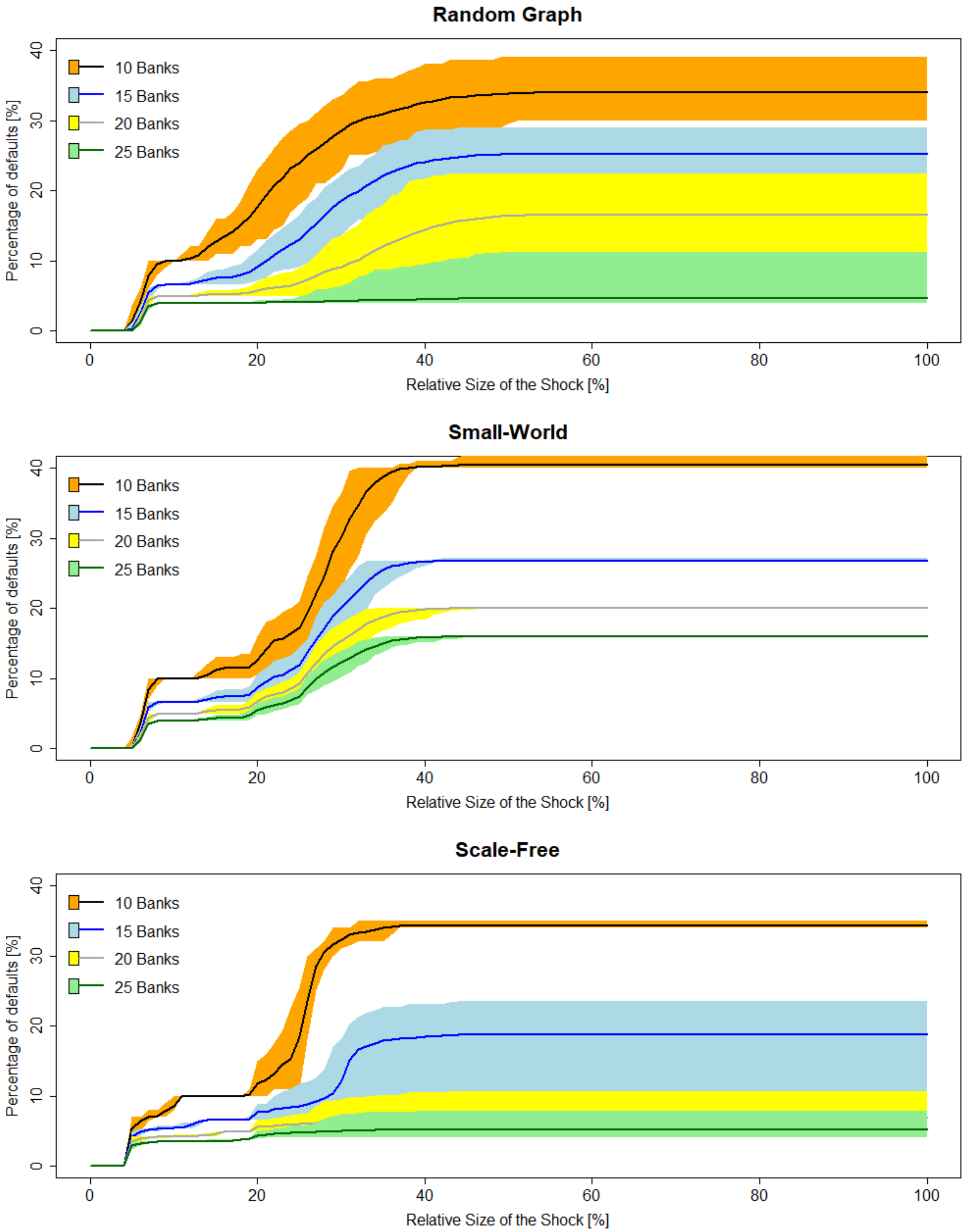

Figure 13: Average percentage of defaults in relationship to shock size for different number of banks and for different network topologies. Following Nier et al. (2007) Fig. 4.

A.4 Additional figures – liquidity risk

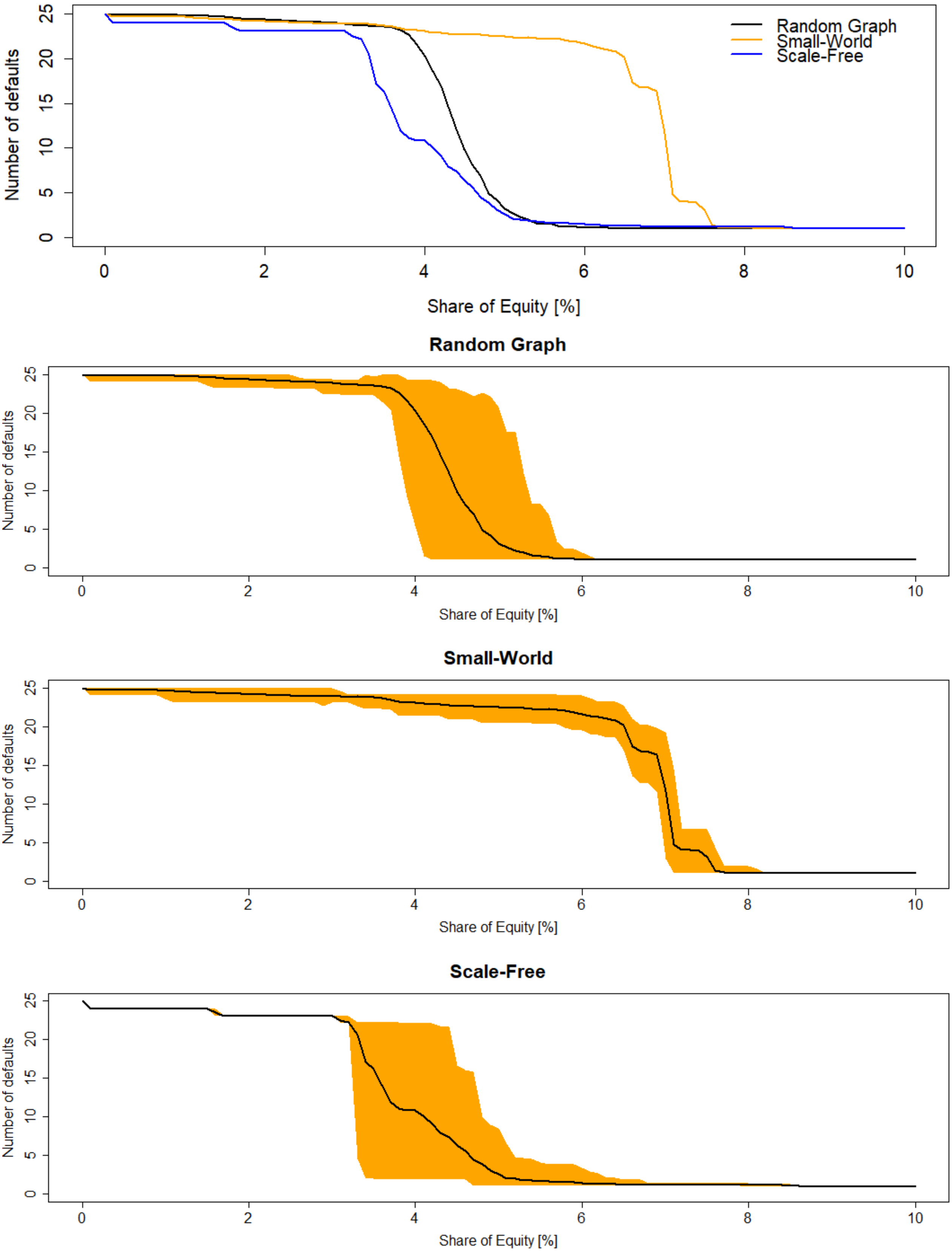

Figure 14: Average number of defaults in relationship to the share of equity to total assets for different network topologies – model with liquidity risk.

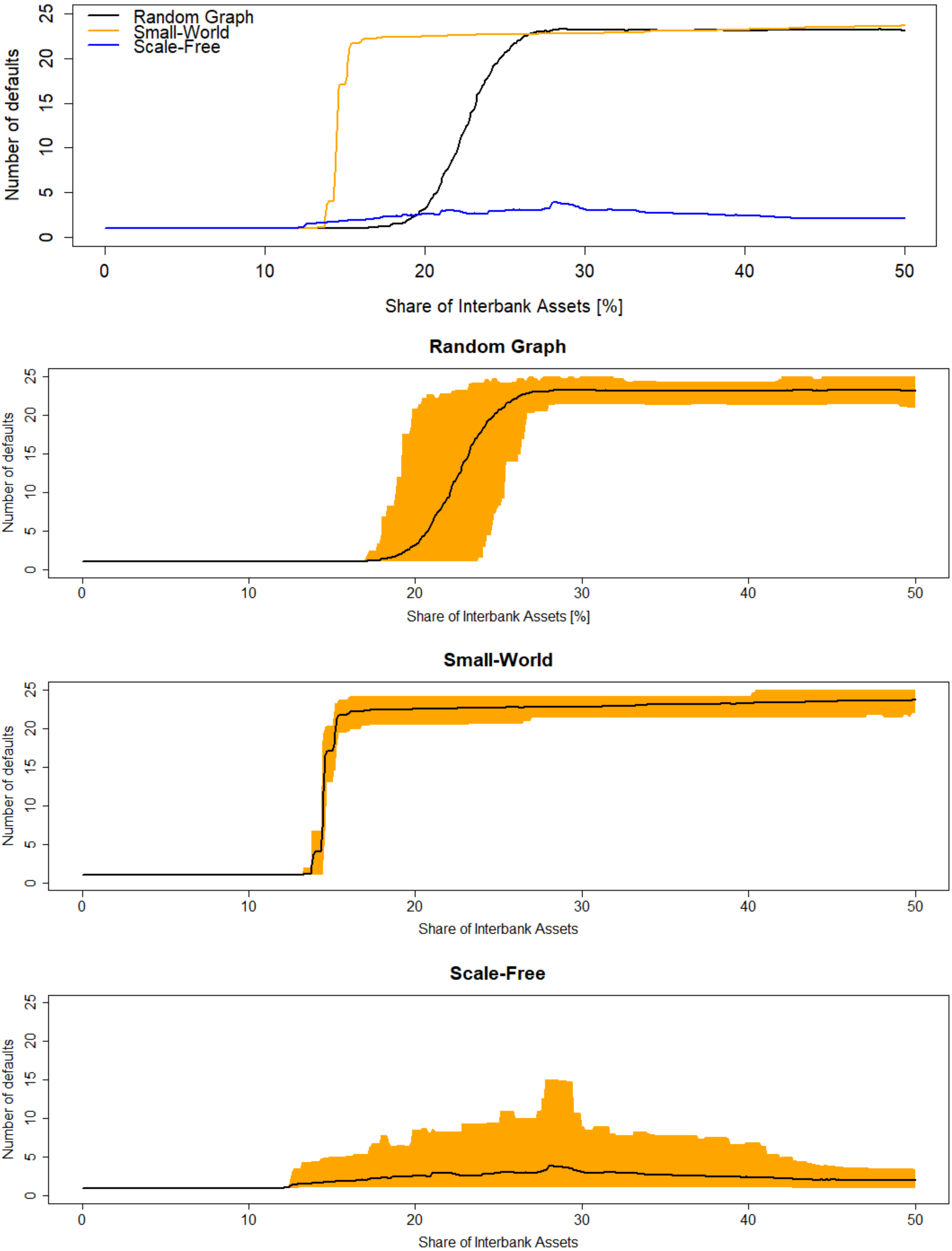

Figure 15: Average number of defaults in relationship to the share of interbank assets to total assets for different network topologies – model with liquidity risk.

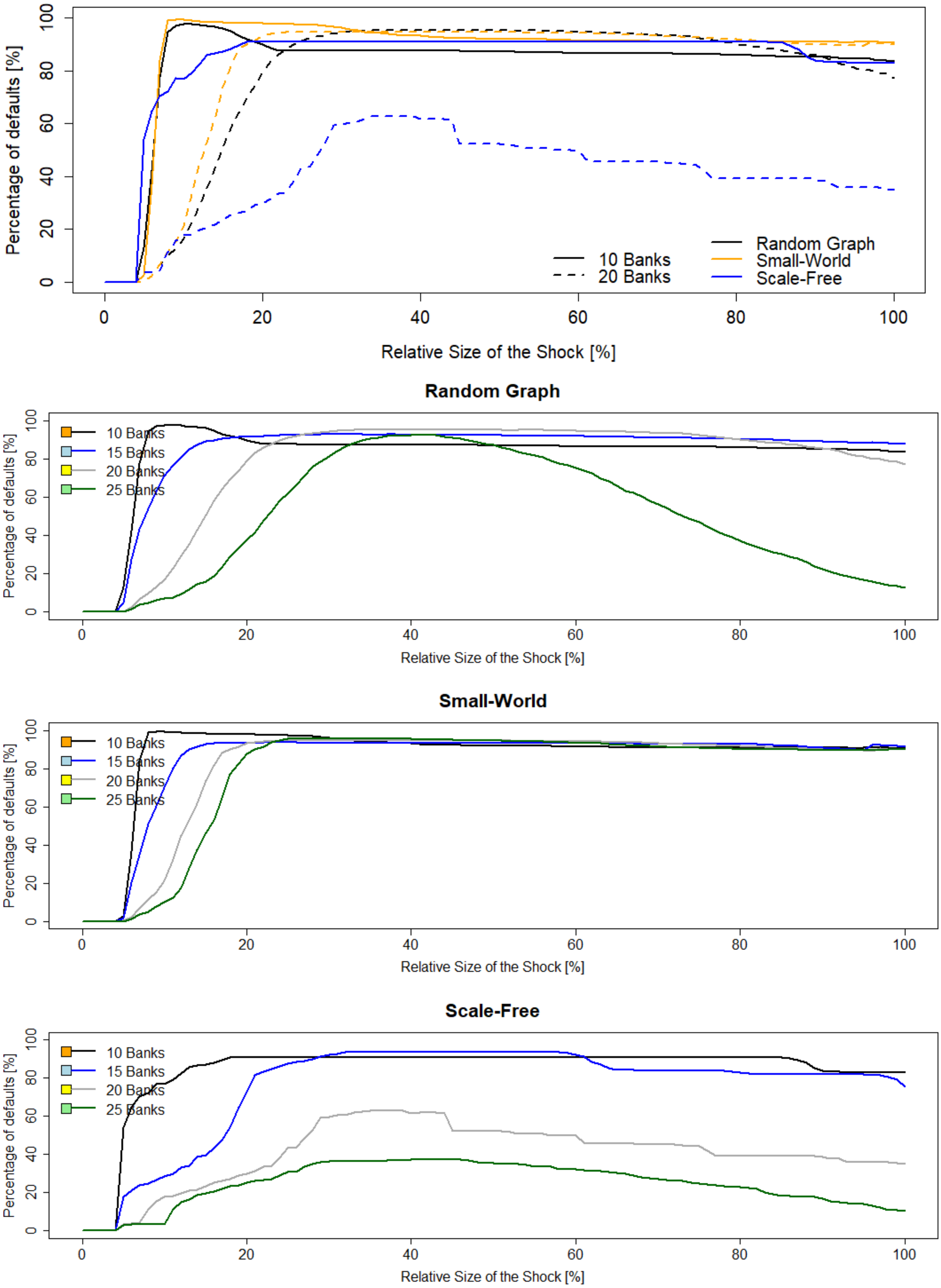

Figure 16: Average percentage of defaults in relationship to shock size for different number of banks and for different network topologies – model with liquidity risk. Range of observations left out for visual reasons.

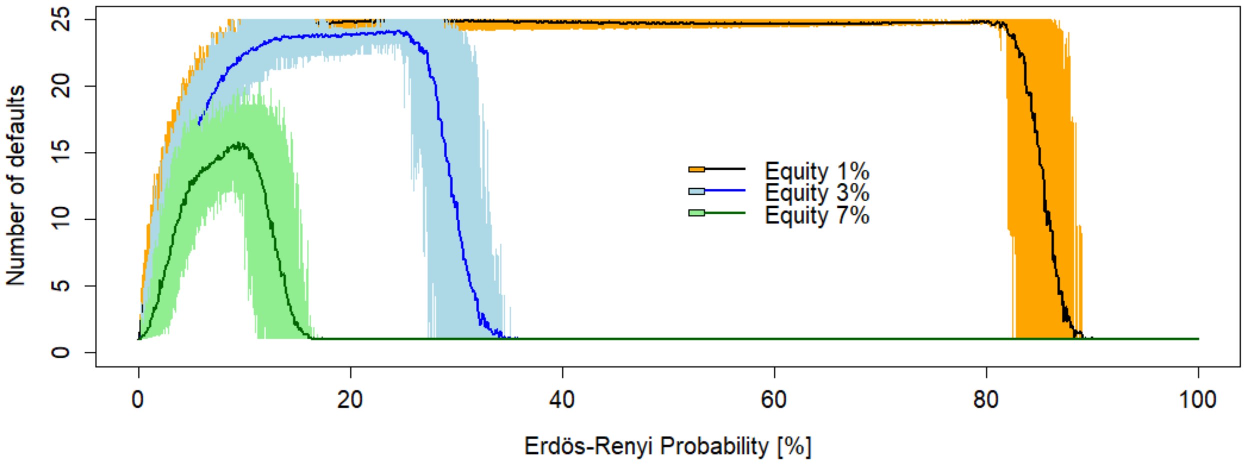

Figure 17: Average number of defaults in relationship to the Erdös-Rényi Probability – model with liquidity risk.

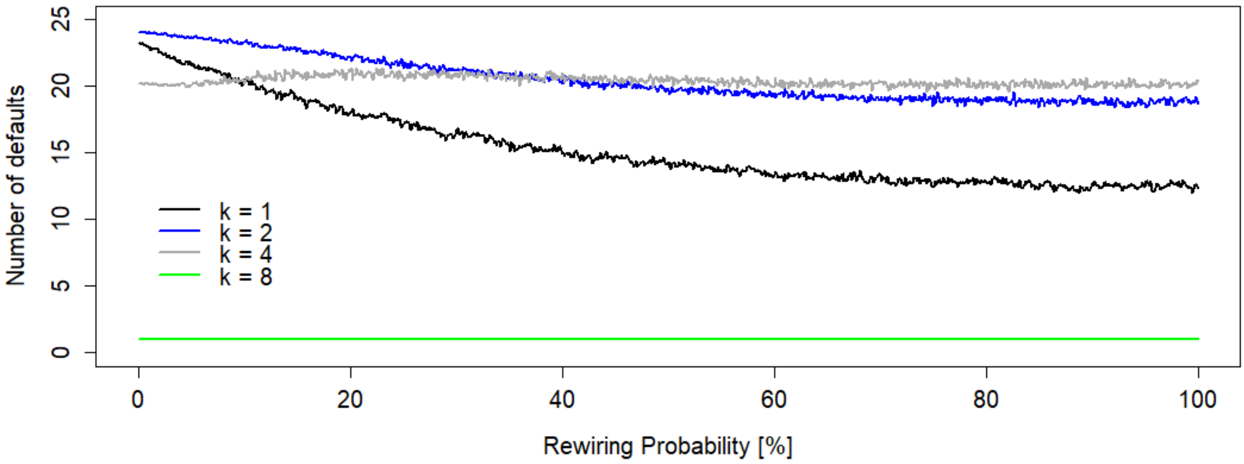

Figure 18: Average number of defaults in relationship to the rewiring probability of small-world networks – model with liquidity risk.

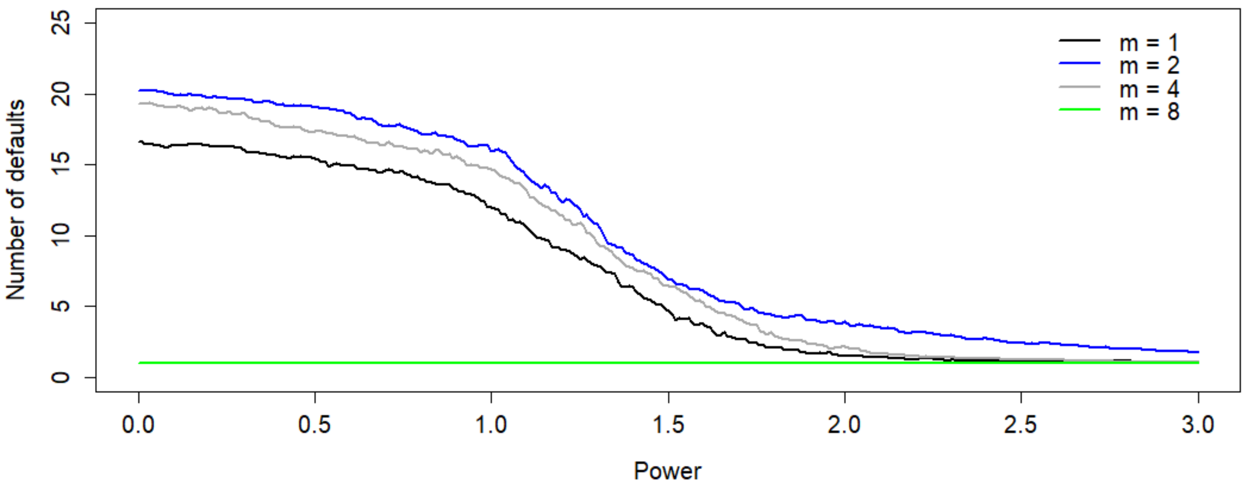

Figure 19: Average number of defaults in relationship to the degree of power of the power-law function of scale-free network – model with liquidity risk.